International Symposium
on
DEFORMATION CHARACTERISTICS
and MODELING of MATERIALS

in honor of

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60th Anniversary

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Sendai International Center
Sendai JAPAN
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Special thanks to
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SESSION 1

Micromechanics Modeling of Composites
Upper and Lower Bounds for the Overall Response of an Elastic/plastic Composite

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Talbot and Willis (1992) presented a general formulation from which all bounds for nonlinear composites known at that time could be obtained. It contained as special cases the original “nonlinear Hashin-Shtrikman” method, which Talbot and Willis (1985) introduced by generalizing the variational approach invented for linear problems by Hashin and Shtrikman (1963), and also the method of Ponte Castañeda (1991), which employed the device of a “comparison linear composite”.

A major limitation of the formulation of Talbot and Willis (1992), and all others, was that it required, for its implementation, the introduction of a linear comparison material – either uniform or a composite – and this permitted the construction of only one bound. For the type of composite considered here, the bound would be a lower bound for the flow potential; thus, the only known upper bound remained the “Sachs bound”, obtained by substituting into the analogue of the complementary energy principle a uniform stress field, as proposed originally by Sachs (1928). Progress with the upper bound problem requires the use of a nonlinear comparison medium, but this introduces an additional “penalty” that needs to be controlled. Talbot and Willis (1995) succeeded in bounding this penalty, by recognising that the trial fields most generally employed have the property of “bounded mean oscillation” (BMO), and developed a direct analogue of the Hashin-Shtrikman upper bound by suitable adaptation of their original formulation of 1985. Subsequently, Talbot and Willis (1996) exploited the BMO technology in the context of their more general formulation of 1992. This permitted the construction of a new upper bound, of third order, which contained a three-point statistical parameter introduced by Milton (1981). Although the reasoning contains one additional layer, the remaining computation was simpler than that required by the earlier implementation. Also, remarkably, it provided a better two-point bound, obtained, in the case that the three-point parameter was unknown, by replacing it by its “worst-possible” value.

All of this is explained in the context of an elastic/power-law plastic matrix, reinforced by elastic particles, both assumed incompressible, and the new upper bound for the flow potential of this composite is given.

References

Improved Self-consistent Estimates for Nonlinear Composite Materials

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This work is concerned with a new procedure (Ponte Castañeda, 1996) for estimating the effective behavior of nonlinear composite materials. The key idea is to replace the nonlinear potentials for each phase by their quadratic Taylor approximations, evaluated at appropriate estimates for the average strains in their respective phases. This allows the conversion of the problem for the nonlinear composite into a problem for a linear thermoelastic composite with anisotropic phases characterized by their “tangent” modulus tensors, and with microstructure identical to that of the nonlinear composite. For two-phase nonlinear composites, the use of Levin’s (1967) result permits further simplification involving only the effective modulus tensor of a linear-elastic comparison composite. The procedure is applied to composites with two isotropic phases and with statistically isotropic particulate microstructures, by taking advantage of the self-consistent estimates of Budiansky (1965) and Hill (1965) for the relevant linear comparison composites. The results are further specialized to composites with power-law constitutive behavior, and explicit estimates of the self-consistent type are obtained for porous materials and particle-reinforced composites. Corresponding estimates are also obtained for transverse and longitudinal (2-dimensional) shear loading of a fiber-reinforced composite with a statistically isotropic distribution of fibers in the transverse plane.

It is found, by construction, that the new nonlinear estimates of the self-consistent type are exact to second-order in the phase contrast, and thus are consistent with the asymptotic results of Suquet and Ponte Castañeda (1993) for weakly inhomogeneous materials. In this sense, the new estimates constitute a more appropriate nonlinear extension of the self-consistent estimates for linear composite materials, which are also known to be exact to second-order in the phase contrast, than the corresponding nonlinear estimates of the self-consistent type of Ponte Castañeda (1991), which are only exact to first order in the contrast. However, unlike these earlier estimates (see also Talbot and Willis, 1985), which could be shown to deliver rigorous bounds, in at least one direction, for the effective potentials of the nonlinear composites, the new estimates do not appear to provide bounds in either direction. Nevertheless, comparison of the new estimates with exact numerical results for rigidly reinforced and more general two-phase composites suggests that the new method may provide some of the most accurate estimates available to date for the effective behavior of nonlinear composites. In particular, the new estimates are the first general estimates of their type to be able to distinguish between axisymmetric tension and simple shear loading of nonlinear composites with isotropic phases and statistically isotropic particulate microstructures. The associated dependence of the new estimates on the determinant of the strain (or stress) is found to become progressively more important in the limit of perfectly plastic behavior, the underlying physical mechanisms explaining these differences in overall behavior having already been identified by Drucker (1966).
References


Bounds on the Overall Properties of Composites with Debonded Frictionless Interfaces

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Damage phenomena at the interfaces between the matrix and the inclusions (e.g. sliding, debonding etc.) considerably modify the overall mechanical properties of the composite materials. Many studies on this subject can be found in the literature (e.g. [1, 4, 5, 7]). The analysis of a composite body damaged by debonding at the interfaces leads to an elastostatic problem which is strongly nonlinear because of the presence of the incompentetribility constraint between the matrix and the inclusions [3]. The main difficulty in searching for the solution of debonding problems is that the actual contact region of debonded interfaces is not a priori known. These problems can be formulated either as variational inequalities or as minimizations of the potential [or complementary] energy of the body on the convex set of the admissible displacement [or stress] fields [6, 8].

In this paper, the homogenization problem is formulated for composites with periodic microstructure characterized by a parallelepipedic unit cell $V$ of volume $\mu$ with a finite number of debonded interfaces, whose union is denoted by $S$. Let $n$ be a unit vector field normal to $S$. Let $H$ be the set of the symmetric tensor fields which are $V$-periodic and square-integrable on $V \setminus S$. Moreover, let $I$ be the set of vector fields defined on $S$, which are $V$-periodic and square-integrable over $S$. It is pointed out that $H$ and $I$ are Hilbert spaces endowed by the usual scalar product.

The strain $E$ is defined as the couple $(F, \delta) \in (H \times I)$, where $F$ has the meaning of strain in $V \setminus S$ and $\delta$ is the displacement discontinuity across $S$. Analogously, the stress $T$ is defined as the couple $(R, \tau) \in (H \times I)$, where $R$ is the stress in $V \setminus S$ and $\tau$ is the stress interaction across $S$. The overall mean strain $<E>$ and mean stress $<T>$ are defined as [9]

$$
<E> = \frac{1}{\mu} \left[ \int_V F \, dv + \int_S \text{sym} (\delta \otimes n) \, ds \right]
$$

$$
<T> = \frac{1}{\mu} \int_V R \, dv
$$

where $\otimes$ denotes the tensor product.

The spaces of the compatible strain fields and the self-equilibrated stress fields are denoted by $E$ and $T$ respectively and are defined as

$$
E = \left\{ (F, \delta) : \exists u \in H^1(V \setminus S) \cap L^2(V) : u \text{ is } V\text{-periodic, } \exists \tilde{F} \in \text{Sym} : F = \tilde{F} + \text{sym} \nabla u, \ \delta = \gamma^+(u + F[x]) - \gamma^-(u + F[x]) \right\}
$$

$$
T = \left\{ (R, \tau) : \text{div} R = 0, \ \tau = \pi^+(R)[n] = -\pi^-(R)[n] \right\},
$$

where $\gamma^+, \pi^+$ and $\gamma^-, \pi^-$ are the trace operators on the oriented surfaces $(S, n)$ and $(S, -n)$, respectively.
The free energy $\Phi$ of the unit cell has the expression:

$$\Phi(E) = \int_{V_{\mathcal{S}}} \phi(F) \, dv + \int_{\partial S^-} I_{\mathcal{S}}^{-} (\delta \cdot n) \, ds,$$

(3)

where $\mathcal{S}^-$ is the set of the negative real numbers, $\phi$ is the free-energy density of the material, and the indicator function $I$ [2] is introduced to impose the incompressibility condition at the debonded interfaces.

The complementary energy $\Psi$ of the unit cell is defined as the Fenchel conjugate of $\Phi$:

$$\Psi(T) = \int_{V_{\mathcal{S}}} \psi(R) \, dv + \int_{\partial S^+} I_{\mathcal{S}}^{+} (\tau \cdot n) \, ds + \int_{\partial \Omega} I_{\mathcal{S}}^{0} (\tau \times n) \, ds,$$

(4)

where $\psi$ is the complementary-energy density of the material, and the indicator functions $I$ are introduced to impose the unilateral frictionless contact at the debonded interfaces.

The homogenization problem can be formulated as the search of the infimum of the free energy potential $\Phi$

$$\bar{\phi}(E) = \frac{1}{\mu} \inf_{E} \Phi(E),$$

(5)

where $\mathcal{E}$ is the subset of $\mathcal{E}$ whose elements have average $E$. It is worth noting that the functional $\bar{\phi}$ is the homogenized free energy density of the body. The homogenized constitutive equation, which relates the average strain to the corresponding average stress $\bar{T}$, can be written as

$$\bar{T} \in \partial_{\mathcal{E}} \phi(E),$$

(6)

where $\partial$ is the subdifferential operator.

On the other hand, the homogenization problem can formulated as the search of the infimum of the complementary energy potential $\Psi$

$$\bar{\psi}(T) = \frac{1}{\mu} \inf_{T} \Psi(T),$$

(7)

where $\mathcal{T}$ is the subset of $\mathcal{T}$ whose elements have average $T$. It is worth noting that the functional $\bar{\psi}$ is the homogenized complementary energy density of the body and the homogenized constitutive equation can be written as

$$\bar{T} \in \partial_{\mathcal{T}} \psi(T).$$

(8)

The issue of consistency between the homogenized constitutive equations (6) and (8) follows by the conjugancy between $\bar{\phi}$ and $\bar{\psi}$, easily proved by convex analysis arguments.

It is worth noting that the overall constitutive behaviour of the heterogeneous periodic material examined in this work is rate- and path-load independent.

When a linearly elastic material is considered, the densities $\phi$ and $\psi$ are quadratic functions. It is shown that the homogenized densities $\bar{\phi}$ and $\bar{\psi}$ are generally not quadratic, but are at least positively homogeneous of degree two. Hence, the homogenized constitutive equations are generally not linear, but they are at least
positively homogeneous of degree one. From a mechanical point of view, this means that the overall constitutive behaviour is multimodular.

Finally, approximation of the previous variational principles are developed by using the finite element method. An iterative procedure is proposed to solve the approximated problem which is non linear for the incompentrability conditions.

References

SESSION 2

Smart Materials and Structures
Analysis on Smooth Growth of Cracks in Brittle Materials

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1. Background

After the great Hanishin earthquake, more attentions are being paid on earthquake faults. Earthquake faults are regularly arranged shear cracks which are formed on grounds when a source fault in the crust extends to the top ground surface. It is necessary to evaluate possible strong motion and large deformation due to the earthquake fault formation, for the design of civil structures of longer service, such as a nuclear waste deposit site.

Riedel shear experiments serve as a model experiment of earthquake faults. It is shown that among numerous cracks which are initiated, only some continue to propagate as loads increase. One may expect that the bifurcation in cracking plays a key role in the formation of earthquake faults. Therefore, the prediction of disaster due to earthquake faults requires an analysis method which is capable to simulate the propagation process with bifurcation.

2. Objectives

A key issue in the analysis of growing cracks is to determine a path along which a crack propagates. This is because, owing to advanced numerical computation techniques, field variables can be computed to any desired accuracy once the crack configuration is specified. The authors are proposing a rigorous formulation of a crack path which uses the change in field variables due to the crack extension. The objectives of this paper, therefore, are to develop an analysis method for growing cracks that is based on this formulation, and to verify the validity through numerical simulation.

3. Formulation of Problem

For simplicity, we consider a crack $\Omega$ which grows in a two-dimensional elastic body $V$; $\pi_i$ and $f_i$ are prescribed on $\partial V$ and on $\Omega$, respectively, and $\Omega$ propagates as $\pi_i$ increases, satisfying the energy release rate criterion, $G = G_c$.

It is shown that the rate of displacement change with respect to the crack extension satisfies

$$
C_{ijkl} \dot{u}_{k,li} = 0 \quad \text{in} \ V \setminus \Omega,
$$

$$
\dot{u}_i = 0 \quad \text{on} \ \partial V, \tag{1}
$$

$$
\dot{t}_i = 0 \quad \text{on} \ \Omega \setminus C^-,
$$

$$
\dot{t}_i = P_i \quad \text{at} \ C^-,
$$

where $C^-$ is the inner crack tip and $[P_1, P_2]$ are given as $\lim_{b \to 0} \sqrt{\pi/8b}[K_{I}, K_{II}]$. These forces are obtained by taking perturbation of traction acting at the extended crack.
It is further shown that the rate of stress intensity factors can be expressed in terms of stress rate associated with $\dot{u}_i$ as

$$\begin{bmatrix} \dot{K}_I \\ \dot{K}_{II} \end{bmatrix} = -\kappa \begin{bmatrix} K_{II} \\ K_I \end{bmatrix} + \lim_{r \to 0} \sqrt{2\pi r} \begin{bmatrix} \sigma_{22,1} + \dot{\sigma}_{22} \\ \sigma_{12,1} + \dot{\sigma}_{22} \end{bmatrix},$$ (2)

where $\kappa$ is the curvature of the extension. Using this relation, we can determine the configuration of the crack extension for a given increment of external loads.

4. Results and Discussion

The growth of a smooth crack can be simulated in an incremental manner according to the above formulation. It should be pointed out that stress rate has singularity of $O(r^{-3/2})$ at the crack tip. This paper uses complex potentials which account for this singularity. The change in crack orientation during the propagation is given by $kappa$, and we apply a condition for the maximum energy release rate to determine $\kappa$. While $\dot{K}_I$ or $\dot{K}_{II}$ is linear to $\kappa$, the second derivative becomes a polynomial of the second order for $\kappa$, and hence $\kappa$ that maximizes $G$ can be found.

It was analytically shown that the proposed formulation is valid for a crack which grows straight. Therefore, we first verify the validity of the proposed formulation when a curved crack grows. The boundary element method is employed to solve boundary value problems for displacement rate. Using a finite difference technique, displacement rate and stress intensity factor rate are numerically computed for various configuration of cracks. It is shown that solutions of the boundary value problems and the resulting stress intensity factor rates coincide with those numerically computed ones.

Next, we seek to reproduce experimental observation of the crack growth using the proposed analysis method. Two different sources of experiments are used; clay and sand samples are used, and a crack (or shear failure) is generated by lifting the bottom of the sample. In both cases, the simulated configuration is in good agreement with the observed one, if material parameters are suitably chosen; in particular, crack configuration such as the dip angle near the top surface is in consistent with experimental measurement. While more realistic modeling is necessary, it is well demonstrated that the proposed analysis method can be applicable to simulate the growth of earthquake faults.

References

Development of Inflatable Rigidized Structures to Adapt Future Space Applications

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Inflatable structures are expected to be a promising candidate for future space structures, because of their compactness for launch, their simple feature for deployment, and adaptivity in design. In this paper, three groups of space rigidized inflatable structures are discussed; they are hybrid structures of membrane and cable network (IRIS, inflatable rigidized integrated structures), space foaming structures, and metal foil structures. A demonstration model of IRIS, which is for good structural accuracy, is presented. Some inflatable and rigidized foam columns and cylinders are also introduced, and their fundamental properties for future practical use are shown.
Micromechanics Modeling of Smart Composites

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“Smart composites” should be distinguished from ordinary composites which are for its primary use as a structural material with high specific mechanical properties. Definition of a smart composite is that it can exhibit a desired function in given environment such as control of a desired shape, induction of desired internal stress and strain. The key element for designing such a smart composite is to use “smart material” as a reinforcement which exhibits coupled behavior where the coupling takes place between any combination of mechanical, thermal and electromagnetic behavior. In this talk, coupled behavior of various smart materials will be stated first, followed by micromechanics modeling of several smart composites which consist of a smart materials as the key constituent and matrix material.

Table 1: Coupling Behavior

<table>
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<tr>
<th>Coupled Behavior</th>
<th>Flux Vector or Field Vector $\Sigma$</th>
<th>Field Vector or Scalar $Z$ or $\theta$</th>
<th>Coupling Coeff. $E$</th>
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<tbody>
<tr>
<td>piezoelectric</td>
<td>stress, $T$</td>
<td>electric field, $E$</td>
<td>$e$</td>
</tr>
<tr>
<td>thermoelastic</td>
<td>strain, $S$</td>
<td>temp. change, $\theta$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>pyroelectric</td>
<td>electric displ., $D$</td>
<td>temp. change, $\theta$</td>
<td>$p$</td>
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<tr>
<td>magnetoelastic</td>
<td>stress, $T$</td>
<td>magnetic field $H$</td>
<td>$k$</td>
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<tr>
<td>magnetoelectric</td>
<td>electric displ., $D$</td>
<td>magnetic field $H$</td>
<td>$\xi$ non-linear</td>
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<td>magnetization, $M$</td>
<td>strain, $S$</td>
<td>$R$ (non-collinear)</td>
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<td>Hall effect</td>
<td>electric current density, $j$</td>
<td>electric field, $E$</td>
<td>$\sigma$ (non-linear in $S$)</td>
</tr>
<tr>
<td>piezoresistivity</td>
<td>stress, $T$</td>
<td>electric field, $E$</td>
<td>$T = f(S, \theta)$</td>
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<tr>
<td>shape memory effect</td>
<td>strain, $S$</td>
<td>temp. change, $\theta$</td>
<td>non-linear</td>
</tr>
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Table 1 summarizes various types of coupled behavior between mechanical, thermal and electromagnetic phenomena. The first five rows denote linear coupling behavior between flux vector $\Sigma$ and field vector $Z$ or scalar $\theta$ while the last four are for non-linear or non-collinear coupling behavior. We have recently established the micromechanics modeling of a smart composite with linear coupling behavior by extending the Eshelby’s model for uncoupled behavior where an example of piezoelectric composite was used. The prediction of coupling coefficients of the composite explain well the experimental results. The Eshelby’s model can also be applicable to the case of a composite with shape memory alloy (SMA) fibers. We have predicted the compressive stress in the matrix material of SMA composite which was given initial prestrain $\epsilon_T$ at room temperature and then subjected to temperature increase beyond austenitic finish temperature ($A_f$). This compressive stress was a dominant contributor to enhance the tensile properties of TiNi SMA fiber/Al matrix composites (stress-strain curve and fatigue resistance) and of TiNi SMA fiber/epoxy matrix.
Piezoresistivity can be observed in any conductive and semi-conductive material that can undergo large elongation. Recently we examined both experimentally and theoretically the piezoresistivity of conductive short fiber/elastomer matrix composite where two analytical models are constructed, fiber percolation model and finite deformation model based on affine transformation. The analytical models are used to explain well the experimental results observed on conductive short fiber/elastomer composite, Fig. 1, where logarithmic of electrical conductivity along $x$-axis, $\sigma_x$, is plotted as a function of $n$-plane strain. Fig. 1 demonstrates clearly piezoresistivity which can be termed as a switchable material.
SESSION 3

Modeling of Inelastic Behavior-I
Intelligent Modeling of Materials

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For several tenths years, many contributions have been made to understand the physics and to reach modeling of aggregates particularly for metals and composites. Prof. S. Nemat-Nasser as well as other important researchers improved and are still improving a lot our knowledge in this fundamental area.

In this paper, we want to show our own view of the problem based on works which were done from 1964 until now.

In the first part, we shall underline what the **physical quantitative** description of inelastic behavior of single crystals, means for us by answering to the following questions:

- are the various glides on each crystal slip plane, potential **internal** variables?
- can we build a **real** physical model for the crystal in correlation the observations of the evolutions of the crystalline defects, such as the dislocations, the vacancies, the precipitates... and the thermal activated processes?
- can we define the **physical** hardenings and is the **latent** hardening higher than the **self**-hardening?
- is it possible to reach a description which gives a **real fitting** with the experimental results?

Then, in the second part, assuming a given behavior for the crystals, for the global inelastic behavior of polycrystals, we shall try to answer to the questions

- how many different orientations of crystals is it necessary to take to reach the description of an initially isotropic polycrystal?
- are the simplified models, such as the self-consistent model, sufficient to represent the interactions between crystals?
- are the actual based on rate-dependant or rate-independent crystals plasticity numerical simulations, sufficient for industrial applications?

At last, in the third part, we shall give our new general framework of “intelligent” modelling of aggregates where we take into account not only the local behaviors of the crystals but also their size, their shape, their relative distribution...

In this framework, it is needed:

1. to build a DATA BASE i.e. to obtain some experimental, real or simulated results where the EXPERTS indicate all variables or descriptors which may take a part.
This is, at first, done with PRIMITIVE descriptors $x$ which are usually in a limited number.

Then, the data are transformed into INTELLIGENT descriptors $XX$ in a larger number with the actual whole knowledge thanks to (but insufficient) beautiful theories!! The descriptors may be number, boolean, alphanumeric, name of files which give access to data base, or curves, signals and pictures treatments. The results or conclusions may be classes (good, not good, ...) or numbers (Young modulus, cost, weight, life time...).

Usually, the data base may contain $\approx 20$ to $50$ examples with $10$ to $1000$ descriptors and $1$ to $20$ conclusions.

2. to generate the RULES with any Learning tool. The Intelligent descriptors help the learning algorithms. Each conclusion is explained as function or rules of some input intelligent descriptors with known Reliability. If this reliability is too low, it implies that whether there is no enough data or there are bad or too many intelligent descriptors are missing.

3. to optimize at two levels (Inverse Problem)

- Considering the intelligent descriptors as independent and it is possible to get the OPTIMAL SOLUTION satisfying the special required properties and allowing the DISCOVERY OF NEW MECHANISMS,
- Considering, the intelligent descriptors linked to primitive descriptors and it is possible to obtain the optimal solution which is technological possible.

So, not only a Practical Optimal Solution is obtained but also the Experts may learn the missing parts, may built models or theories based on the kept intelligent descriptors and guided by the shapes of the rules or relationships.

We shall illustrate our framework by treating step by step the problem of the global elastic behavior of concrete with a soft cement matrix and hard inclusions. The problem was already the object of several papers; simplified models, simple bounds of the elastic modulus and sophisticated theories... were produced. Our aim to give the results to the engineers in such a practical way that they could:

- foresee the elastic properties within a few percents of error for any concentration, any kind of shape or distribution of the inclusions,
- select the preparation of the aggregate to reach any elastic properties which may be given in advance.
Formulation of Constitutive Law for Frictional Materials

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A basic framework is proposed for the constitutive modeling of frictional materials. The theory has capability to reflect dissipation mechanism in terms of the dissipation function. The dissipation function in this theory expresses the energy density required to produce unit dissipative strain. The dissipation condition, which corresponds to yielding condition in the plasticity theory, is given by the equilibrium equation for dissipation energy. The flow rule is derived from the condition that the equilibrium equation for dissipation energy is attained when the difference between the dissipation energy and the corresponding dissipative work becomes minimum. The hardening law is derived in the same manner as the plasticity theory based on the dissipation condition.

As the plasticity theory, the strain-rate consists of the elastic and dissipative strain-rates;

\[ \dot{\varepsilon} = \dot{\varepsilon} + \dot{d} \]

where \( \dot{\varepsilon} \) is the elastic strain-rate and

\[ \dot{d} = n\dot{\alpha}, \quad n : n = 1, \quad \dot{\alpha} = \sqrt{d : d} \]

is the dissipative strain rate. The parameter \( \alpha \) increases monotonically as the dissipative deformation progresses. Such a phenomenon as the equivolumetric deformation in the metal plasticity or the dilatancy in the geomaterials causes the restriction between components of dissipative strain-rate, and it may be expressed in the following form;

\[ l : n = C, \quad l : l = 1 \]

where \( l = l(\sigma, \alpha) \) and \( C = C(\sigma, \alpha) \) are given functions.

The inner product between the stress and the dissipative strain-rate gives the rate of dissipative work, and the dissipative work per unit dissipative strain is defined as

\[ \omega \equiv \sigma : n. \]

Just as the energy release rate in the fracture mechanics, there may exist a certain level of energy which enables the dissipative strain to occur. The dissipative function

\[ \varphi = \varphi(\sigma, \alpha, n) > 0 \]

expresses the level of energy which is required to produce the unit dissipative strain in the direction of \( n \), and its value should be always positive by the second law of thermodynamics.

The dissipation condition is the equilibrium equation for dissipation energy given as follows.

\[ f \equiv \varphi - \omega = 0 \]

If this equation is not fulfilled for all the direction \( n \), then the deformation is elastic. In the elastic state \( \omega \) does not reach the level \( \varphi \), and an inequality \( \varphi - \omega > 0 \) holds.
When $\omega$ reaches $\varphi$ for the first time, then $f$ takes minimum value 0 for a certain direction $n$. Thus we can determine $n$ by the following condition:

$$\frac{\partial F}{\partial n} = 0, \quad F = f + \frac{a}{2} n : n + b l : n$$

where $a$ and $b$ are Lagrangian multipliers, or explicitly we obtain the following flow rule:

$$n = \frac{\tau - bl}{a}$$

where

$$\tau \equiv \sigma - \frac{\partial \varphi}{\partial n}, \quad a = \sqrt{\tau : \tau - (\tau : l)^2 \over 1 - C^2} \quad \text{and} \quad b = \tau : l - \frac{C \sqrt{\tau : \tau - (\tau : l)^2}}{\sqrt{1 - C^2}}.$$

As can be seen, the flow rule takes rather complicated form when $\varphi$ is the function of $n$.

The dissipation condition keeps on holding with the consistency condition:

$$\dot{j} = \frac{\partial \varphi}{\partial \sigma} : \dot{\sigma} + \frac{\partial \varphi}{\partial d} : n \dot{\alpha} - n : \dot{\sigma} - \tau : \dot{n} = 0$$

which leads to the hardening rule

$$\dot{\alpha} = \frac{m : \dot{\sigma}}{D}$$

where

$$m \equiv n - \frac{\partial \varphi}{\partial \sigma} + b \left( \frac{\partial C}{\partial \sigma} - n : \frac{\partial l}{\partial \sigma} \right), \quad D \equiv \left\{ \frac{\partial \varphi}{\partial d} - b \left( \frac{\partial C}{\partial d} - n : \frac{\partial l}{\partial d} \right) \right\} : n.$$  

Finally we get the incremental relationship between the stress and the dissipative strain:

$$\dot{d} = \frac{nm}{D} : \dot{\sigma}.$$  

The above equation suggests the constitutive relationship is generally non-associative. Together with this equation as well as the elastic relationship for the elastic strain, we get the complete constitutive relationship.

An example in which the suitable functions for $\varphi(\sigma, \alpha)$, $l(\sigma, \alpha)$ and $C(\sigma, \alpha)$ were assumed verifies the availability of this theory to the granular materials.

Generally, one of the most difficult problems in the constitutive theory for materials is how the micro-level behavior is related to the macro-level characteristics. If a chosen dissipation function reflects the micro-level behavior of a material properly, then the constitutive relationship obtained by this theory might become a rigorous constitutive relationship for that material.
Granular Modeling Approach to Electro-packaging Materials

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Introduction

The particulate material to be used for electro-packaging of LSI-chip is a compound of polymer matrix and silica particles. In the materials design, a volume fraction of silica must be designed to reduce the difference of thermal expansion coefficients between the compound and the semiconductive device. In case when the volume fraction of silica is relatively small, the visco-elastic behavior of this compound in process is governed by the rheological characteristics of thermoset polymer matrix. Then, viscous flow of the compound might well be identified as a semi-newtonian fluid. While, with increase of loading ratio of this silica, non-newtonian viscosity is observed and various defects are left in this type of electro-packaging materials after heating process. In order to investigate essential cause of these defects, to predict formation mechanism of these defects and to search for alternative processing free from defects, rheological behavior of the high-loaded particulate materials must be precisely described. New adaptive granular modeling to this type of particulate materials is proposed in this study to understand the effect of particle-particle interactions on the observed non-newtonian behavior.

Granular Modeling

The high-loaded particulate material is represented by an assembly of compound elements. Each compound element is composed of the elastically deforming core-particle element and the visco-elasstically deforming polymer shell. With consideration of mutual interactions among particles, or, between one particle and other soft shells, or, between soft shells, direct solution of the conservation laws of momentum and energy determines rheological response of this particulate material subjected to the externally applied stresses and/or velocity. The calculated quantities are averaged to obtain the apparent strains and stresses comparable to the experimentally measured results. Direct visualization of particles enables us to evaluate a role of local non-newtonian flow on the void formation.

Discussion and Conclusion

Two dimensional visco-elastic behavior subjected to shear stresses reveals that the velocity distribution should be shifted from linear to curved profiles with increasing the volume fraction of silica particles; non-newtonian visco-elasticity is activated by the silica particle interaction. Local agglomeration and separation of particles must be an essential driving mechanism to change the macroscopically measured theological behavior in the nonlinear manner with the loading ratio. Three dimension geometry of the die-set configuration is simplified into two dimensional problem in cross-section to evaluate the visco-elastic behavior at the vicinity of a gate; as have been experimentally observed, a crack or a void is easy to be formed near the
gate. Two dimensional process simulation can directly provide us change of particle rearrangement and its mechanical interaction in the transient flow from a gate into a die cavity. Of great importance is temporal change of viscosity coefficient of thermoset polymer matrix; once a significant void is formed among particles in the intermediate stage, this void configuration might be often frozen in the matrix. Our granular model must be the first trial to clarify the formation process of various defects in the injection molding. Furthermore, authors are ready to make comments on the macroscopic representation of these calculated physical quantities by using the statistical physics and the related homogenization theory.
SESSION 4

Modeling of Inelastic Behavior-II
Microstructural Ductile and Brittle Material Failure Modes in Polycrystalline Materials

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Introduction

Ductile and brittle failure has been investigated for cubic polycrystalline materials separated by random grain boundaries. Thermo-mechanical deformation modes that result in ductile failure, such as shear-strain localization, or brittle failure, such as macroscopic crack formation, have been characterized. Dislocation based theoretical and computational constitutive formulations have been developed to account for the effects of random grain-boundary misorientations, dislocation densities, thermal and geometrical softening, and strain-rate sensitivity on failure evolution.

Formulation

A brief outline of the dislocation based constitutive formulation is presented here. A detailed formulation is given by Zikry and Kao (1995). It is assumed that inelastic components of the deformation-rate tensor and the spin tensor are defined in terms of the crystallographic slip-rates on each slip system. For a rate-dependent formulation, the slip-rates are functions of the resolved shear stresses, the reference stress, and the rate sensitivity. The reference stress on each slip system is given as a function of the immobile dislocation density, rim. The mobile dislocation and immobile dislocation densities evolve on each slip system $\alpha$ as

$$\frac{d\rho_{im}^{(\alpha)}}{dt} = \dot{\gamma}^{(\alpha)} F(g_1, g_2, g_3) \quad (1a)$$

$$\frac{d\rho_{m}^{(\alpha)}}{dt} = \dot{\gamma}^{(\alpha)} F(g_1, g_2, g_4) \quad (1b)$$

where $g_1$ and $g_2$ are related to the immobilization of mobile dislocations, $g_3$ is related to the generation of immobile dislocations, and $g_4$ is related to the recovery of mobile dislocations. Since the deformations considered in this study are essentially adiabatic, the temperature is updated as a function of plastic work.

Numerical Method

The total deformation tensor, and the plastic deformation tensor, are needed to update the stress state of the crystalline material. A brief outline of the numerical method will be presented; for further details see Zikry (1994). To obtain the plastic deformation rate tensor, the time derivative of the resolved shear-stress is used together with an objective stress rate for lattice distortion, to obtain a system of nonlinear differential equations, for the resolved shear-stresses as a function of the deformation-rate tensor, the resolved shear stresses, the rate sensitivity parameter,
the slip-rates, and the current lattice orientations. The solution of this system of differential equations is numerically difficult, not due to numerical stiffness in certain time intervals. To deal with this problem, a finite-element algorithm, based on A-stability, has been used to ensure accurate and stable solutions.

Results and Discussion

The accuracy of the predictive capabilities of the dislocation based constitutive formulation and the finite-element algorithm were investigated by the simulation of experimental results, obtained by Rey and Zaoui (1982), pertaining to the inelastic deformation of isoaxial copper bicrystals. This details of this simulation are summarized in Zikry and Kao (1996). There is good agreement between the numerical simulation and the experimental results. These numerical results are a strong indication that the dislocation-based crystalline constitutive formulation and the finite-element scheme result in accurate predictions of the inelastic deformation of crystalline materials.

References

Important Role of Induced Anisotropy in the Development of Shear band in Granular Materials

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Micro-structural changes taking place in two shear bands of Toyoura and Ticino sands were observed by means of a method of X-rays application and of optical measurements using a microscope and thin sections. Also photoelastic pictures taken from a biaxial test on a two-dimensional assembly of oval rods were analyzed with special interest in the development of micro-structure in shear bands, with the following results:

1. Extremely large voids are produced in shear bands, and the resulting void ratio can be larger than a maximum void ratio determined by the Japanese standard method.

2. Particle orientation changes sharply at shear band boundaries, so that high gradient of particle rotation can be developed within a relatively narrow zone during the shear banding process.

3. In the strain hardening process, the main micro-structural change is setting up of columns extending parallel to a major principal stress direction. The columns start buckling at a peak stress, and the buckling columns tend to concentrate in shear bands during the strain softening process, which causes not only the growth of the extremely large voids but also the particle rotation in shear bands.

Based on these observations, the following two points were suggested:

1. Rotational stiffness at contacts must be one of the important components controlling the strength of granular soils, and

2. couple stress, as a result of couple forces at contacts, must be taken into account in the formulation of static equilibrium condition in a shear band and in the interpretation of residual strength of granular soils.

Numerical tests were carried out to simulate the micro-structure developed in shear bands, as well as overall stress-strain behavior of granular soils. To do the simulation analysis, the conventional distinct element method (DEM) was modified a little such that the effect of the moment transmission through contacts can be taken into account. The results are summarized as follows:

1. The development of shear bands can well be simulated only when the moment transmission is considered in DEM. That is, not only the generation of large voids but also the particle rotation in the shear bands can both be reproduced, in a quite similar manner to those of the natural granular soils.
2. Column-like structure (column) is well developed in DEM, regardless of whether the moment transmission is considered or not. This result supports strongly the idea that the development of the columns is a common feature in the strain hardening process of granular soils. In the following strain-softening process, buckling of the columns takes place in parallel with the development of shear band. Generation of large voids and particle rotation are produced as a result of the buckling of the columns.

From the experimental and numerical studies, two conclusions were obtained:

1. Rotational stiffness at contacts plays a key role to develop the micro-structure (large voids and particle rotation) in shear bands.

2. Increasing anisotropy by the column-like structure, which is generated during the strain-hardening process, provides the preliminary stage for strain localization in shear bands after failure. In other words, any clear shear band dose not appear unless the induced anisotropy is developed.
Failure Mechanism and Pull-out Strength of a Bond-type Anchor near a Free Edge

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Introduction
Bond-type anchors fasten the structures to massive concrete or rocks by its bonding strength on the surfaces of anchor bolts. They are so called post-installed systems and have been used often in the retrofit of masonry or concrete structures. The recent development of construction technology has diversified the applicability, and now in some cases they are used in new construction sites. The bond-type anchor, unlike a usual headed anchor, exhibits complex failure modes. If we preclude the trivial failure by the yielding of anchor bolt, the potential failure modes of the bond-type anchor are classified as a) bond failure, b) cone-failure, and c) mixed bond-cone failure depending on material and geometric conditions. (Fig. 1) When the bonding resistance on the bolt surface is small, the anchor fails by pulling out and when the bond between the bolt and the surroundings are perfect, only cone failure is possible and permissible. The usual failure mode, therefore, is somewhere between these two extreme cases, resulting in the complex mixed bond-cone failure. Because of this complexities, the rational design method of the bond type anchor is still to be sought for. In a simple case, we have already proposed the method to estimate the strength and the failure mode. Among the various cases to be contemplated, in this work, we focus the effect of free edge on the pull-out strength with the help of linear fracture mechanics.

![Figure 1: Failure mode of bond-type anchor](image)

The Failure of the Bond-type Anchor near a Free Edge
Let us consider the pull-out strength of bond type located at the distance \( c \) from a free edge under the assumption that the bond stress on the bolt surface is uniform regardless of the proximity of a free edge. As the load increases, the cone failure becomes possible at depth \( x \) when the strength of stress cone and the bonding at depth \( x \) are balanced there. That is, the following equations hold.

\[
P_c(x) = P_b(x) \quad \text{and} \quad \frac{dP_c}{dx} \bigg|_{x=x_c} = \frac{dP_b}{dx} \bigg|_{x=x_c} \quad (1), (2)
\]
where $P_c(x)$ and $P_b(x)$ are the strength of the stress cone and the bond strength at depth $x$. Figure 3 illustrates the schematic expression of Eqs. (1) and (2). Since we assume $P_b(x)$ is independent of $c$, only $P_c(x)$ is affected by the existence of the free edge. As observed in Fig. 3, while the pull-out strength continuously changes, the depth of the stress cone can have a jump under the influence of a free edge and is heavily dependent of the shape of $P_c(x)$ curve. At this moment, the specific expression of $P_c(x)$ itself poses an open problem. For example, ACI suggests the use of the stress cone with uniform stress distribution on the failure surface for headed anchors. That is

$$P_c(x) = A_c(x)\sigma_{cone} \propto x^2$$

(3)

where $A_c(x)$ is the projected area of the stress cone and $\sigma_{cone}$ is the tensile strength of the surrounding material. The effect of a free edge can be estimated according to ACI 349-85. That is, the reduction in pull-out strength is equivalent to the reduction in projected area caused by the proximity of the free edge. Eq. (3) has been accepted in engineering practices. However, it oversimplifies the progressive development of the stress cone before the load reaches the pull-out strength. In fact, many experimental results of headed anchors implied that the dependence of $P_c(x)$ on the depth $x$ is closer to $x^{1.5}$ than $x^2$ especially when $x$ becomes larger. Suggested by these experimental observation, we apply linear fracture mechanics analysis for a bond-type anchor and showed that $P_c(x)$ is proportional to $x^{1.5}$ also in this case. Thus, as alternative of Eq. (3), we propose

$$P_c(x) = Cx^{1.5}$$

(4)

where $C$ is a constant which depends on the fracture toughness, elastic moduli of the surrounding material and the geometry. Since Eq. (4) is based on the consideration of the evolution of a cone-shaped crack, the effect of a free edge on $P_c(x)$ can be accounted for in terms of the arc length of a crack front.

We carried out the simple pull-out test of a bond-type anchor embedded in mass concrete. Figure 4 shows the results with the analytical prediction based on Eq. (3) and Eq. (4). The horizontal axis is the distance to a free edge, $c/a$ (Fig. 2), and the pull-out strength is normalized by the one without the effect of a free edge. The effect of a free edge is much larger than expected from Eq. (3). On the other hand, the prediction by Eq. (4) is very close to the experimental results.

**Figure 2:** Bond-type anchor near a free edge

where $P_c(x)$ and $P_b(x)$ are the strength of the stress cone and the bond strength at depth $x$. Figure 3 illustrates the schematic expression of Eqs. (1) and (2). Since we assume $P_b(x)$ is independent of $c$, only $P_c(x)$ is affected by the existence of the free edge. As observed in Fig. 3, while the pull-out strength continuously changes, the depth of the stress cone can have a jump under the influence of a free edge and is heavily dependent of the shape of $P_c(x)$ curve. At this moment, the specific expression of $P_c(x)$ itself poses an open problem. For example, ACI suggests the use of the stress cone with uniform stress distribution on the failure surface for headed anchors. That is

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Figure 3: Failure mechanism of a bond-type anchor

Figure 4: Pull-out strength of a bond-type anchor near a free edge
Micromechanics-based Continuum Modeling of Geological Materials

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Mechanical behaviors of geological materials including ice are governed by the existence and behaviors of microstructural elements such as joints or microcracks in rock and basal planes in ice grains. To find solutions to engineering problems such as design of a large scale cavern in jointed rock mass, the problem must be set as a mathematical problem, a boundary value problem in many cases. The number of microstructural elements involved in the problem is enormous and it is almost impossible to treat individual elements separately. Hence the original body with microstructures is replaced by an equivalent continuum. The point is how to ensure the equivalence between two problems. “Micromechanics-based Continuum (MBC) Modeling” provides a constitutive equation in the equivalent continuum problem by taking average of strain and stress over a representative volume element. This article introduces several examples of application of MBC modeling to geological materials and demonstrates how successfully MBC modeling answers engineering requests. It should be emphasized that most of engineering problems are governed by evolution of inelastic behaviors of microstructural elements. Therefore, it is not sufficient to consider only the overall modulus, but the change in the microstructure must be treated in the derivation of constitutive equations. Examples of applications are summarized as follows.

Constitutive modeling of ice: A micromechanics based constitutive model is developed to predict the time dependent deformation behavior of ice under compressive load in which creep and microcracking are the two dominant mechanisms. The model is based on the idea that ice crystals undergo shear slip along the basal plane, and the microcracking is caused by the local stress due to the mismatch strain in the polycrystalline ice with randomly oriented basal plane. The model predictions for constant load and constant strain rate tests are shown to demonstrate the validity of the model. The proposed constitutive model is implemented in the finite element analysis code to analyze two-dimensional indentation problems of ice sheet. The present analysis can predict ice forces which act on structures from a moving ice sheet. As an example, a small scale indentation problem is analyzed and results are compared with existing experimental data with qualitative agreement.

Excavation of Jointed Rock Mass: Opening and sliding of joints, which are initially closed by earth pressure, are considered to be the governing mechanism of behaviors of rock masses in the underground excavation. The amount of opening and sliding of joints depends on the constraint by the surrounding rock mass. A micromechanics-based continuum (MBC) model of jointed rock masses is developed based on such mechanisms. The constitutive equation of the equivalent continuum is derived as the relationship between average stress and average strain over a representative volume element which contains a number of joints. The derived constitutive equation reflects effects of density, orientation and connectivity of joints as
well as the property of joints itself. The constitutive equation is implemented into FEM code for the analysis of problems with arbitrary geometry and boundary conditions. The actual excavation problems are analyzed and distribution of opening and sliding displacement of joints as well as displacement and strain of rock masses are obtained. The numerical results are compared with measurement data showing remarkable agreement.

**Simulation of the Hot Dry Rock reservoir formation:** MBC model of jointed rock mass is extended to express behaviors under hydrofracturing. Three dimensional analysis of HDR reservoir stimulation presents the effect of joint orientation, joint spacing and initial stress on the size and shape of the HDR reservoir. The shape of the predicted reservoir is compared with AE source distribution in the actual stimulation tests. The predicted critical water pressure is in accord with reported data.

**Creep Failure of Hard Rock:** A micromechanics-based continuum theory which can reproduce localization phenomena is introduced for creep deformation and creep failure of hard rock. The information important for localization is lost through usual homogenization processes. It is necessary to account for the interaction effect between microstructural elements directly in the evolution law. In the present theory a new field quantity is introduced which represents the interaction effect. The additional governing equation associated with the new field quantity is shown to be an integral equation. Numerical results show that the softening behavior following a peak point and bifurcation from homogeneous deformation to localized one are reproduced by the present theory. Predicted axial strength in short-term failure and failure time in creep tests as a function of confining stress are compared with experimental data showing agreement.
Asymptotic Modelling of Layered Structures

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In this work we present an asymptotic analysis of layered structures, which consist of thin and soft interface layers. A rigorous asymptotic algorithm enables us to study boundary value problems for the Lamé system in thin domains and to derive simplified (by reducing the dimension of the problem) equations of thin compound structures. A particular simple example is related to the derivation of a mathematical model for a thin compound beam that consists of three layers, with the middle one being infinitesimally thin and soft in comparison with the two others; it models an adhesive joint where two elastic layers are bonded together by a thin layer of glue. The limit problem corresponds to an Euler-Bernoulli beam with an internal line of discontinuity for the longitudinal displacement.

A similar problem was considered by Klarbring (1991), where the asymptotic approach was based on a variational formulation developed by Ciarlet (1990). However, that model did not reduce the dimension of the problem.

The asymptotic algorithm, employed in our work, involves two small parameters: first, the normalized width and, second, the normalized Young’s modulus of the adhesive layer. We introduce several classes of compound structures characterized by different relations between these small parameters, and develop an asymptotic algorithm that generalizes the technique of analysis of elliptic boundary value problems posed in thin domains (we refer to the book of Maz’ya, Nazarov and Plamenevskii (1991, 1992), and to the paper by Movchan and Nazarov (1988)) to the case of strongly inhomogeneous elastic structures.

As a result, we obtain a rigorous and simple mathematical model for a laminated structures with a soft adhesive middle layer. We present rigorous derivation of equations in reduced dimensions, and, in particular, the model includes the differential equation for the jump of the longitudinal displacement across the adhesive. The mathematical model exhibits complete agreement with the results available in the engineering literature. The results of this work are summarized in Klarbring and Movchan (1995).

References


SESSION 5

Experimental-Numerical Analysis
**T**$^\ast$-$\epsilon$-integral Analysis of Aluminum Specimens

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$T^\ast_\epsilon$-integral is a path dependent integral based on the incremental theory of plasticity where, unlike the J integral, the integration path elongates with the extending crack. When evaluated very close to the quasi-statically or dynamically extending crack, $T^\ast_\epsilon$ represents the local energy consumed in a strip encompassing the crack tip plastic region and in the wake of the extending crack. The purpose of this investigation is to determine if $T^\ast_\epsilon$ can be considered a fracture parameter which governs crack growth.

$T^\ast_\epsilon$-integral values associated with stable crack growth in 2024-T3 aluminum single edge notched (SEN) and central notched (CN) specimens of 0.8 mm thickness and compact (CT) specimens of 3.1 mm thickness were determined both experimentally and numerically. Experimentally, the $T^\ast_\epsilon$ was computed by evaluating the contour integral along a near-crack contour, $\Gamma_\epsilon$, which elongated with crack extension. This numerical integration, based on the crack displacement field determined by moire interferometry, was conducted along a rectangular contour, $\Gamma_\epsilon$. This $\Gamma_\epsilon$ was located at a distance one specimen thickness from the crack and crack tip in order to assure that the state of plane stress prevailed along the integration path despite the presence of a 100% shear lip. $T^\ast_\epsilon$ was also determined directly by the equivalent domain integration of the results generated by elastic-plastic finite element (FE) simulations of the stable crack growth tests of SEN, CN and CT specimens.

Figure 1 shows the experimentally determined $T^\ast_\epsilon$ values of the SEN, CN and CT specimens with stable crack growth. These $T^\ast_\epsilon$ are in excellent agreement with the corresponding FE results. The good agreement between the $T^\ast_\epsilon$’s of the SEN and CN specimens indicate that unlike the J integral, $T^\ast_\epsilon$ does not vary with geometry. The lower of $T^\ast_\epsilon$ of the CT specimens is due to the increasing effect of the state of plane strain in the thicker CT specimen. Figure 2 shows the crack tip opening angles (CTOA) of the three specimens. Other than the differences in initial blunting, the CTOA’s for the three specimens are essentially identical thus suggesting that CTOA cannot discriminate between the plane stress and plane strain fracture resistance.

The above preliminary findings suggest that $T^\ast_\epsilon$, for a given $\Gamma_\epsilon$, could be used a stable crack growth criterion\(^1\).

\(^1\)The results reported here were obtained through a FAA Research Grant 92-G-005
Figure 1: $T_*^*$ integral for SEN, CN, CT specimens 2024-T3 aluminum alloy

Figure 2: CTOA for SEN, CN, CT specimens 2024-T3 aluminum alloy
Fatigue Limit Evaluation of Metals using an Infrared Thermographic Technique

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1. Introduction

In recent years, fracture mechanics has become the primary approach to controlling brittle fracture and fatigue failure in structures. Since the fatigue is generally caused by the cyclic plastic strain, the plastic strain energy plays an important role in the damage process. Fatigue cracks generally initiate from surface defects or discontinuities and thus predominantly influenced by the surface stress system. The significance of such an energy approach is in its ability to unify microscopic and macroscopic testing data, and subsequently to suggest multiaxial design criteria. Therefore, the idea of relating fatigue to the intrinsic dissipation detected by infrared thermography seems to be highly relevant. This paper emphasizes the application and use of infrared thermography to damage and fatigue evaluation of metallic materials, and subsequently to nondestructive testing on steel structures.

2. Heat Production Mechanism during Fatigue Process

According to Moore and Kommers [1921], the temperature test was suggested and to some extent used by C. E. Stromeyer of Manchester in England as long ago as 1913. However in their searching for some short-time test for predicting fatigue resistance, their technique, based on the use of thermocouples, did not permit to establish thoroughly the temperature limit as identical with the endurance limit under repeated stress.

The question of when a crack does initiate to become a propagating crack, seems to be somewhat philosophical, particularly when searching for some short-time test for determining fatigue limit. This is found by using the infrared thermographic technique to quantitatively evaluate the rise of temperature under a reversed stress applied for a few minutes or less. Infrared thermography readily detects the occurrence of both initiation and propagation of failure. In addition, this technique describes the failure location and process of the structure failure.

Infrared thermography has been successfully used as an experimental method to detect the plastic deformation of a steel plate under monotonic loading (Bui et al [1987]) or as a laboratory technique for investigating damage, fatigue and failure mechanisms occurring in engineering materials and structures (Luong [1990]).

3. Infrared Thermographic Technique Background

The development of thermo-elastic-plasticity equations requires three types of basic assumptions (Allen [1985], Dillon [1963], Kratochvil and Dillon [1969], Taylor and Farren [1925]):

- The basic thermomechanical quantities describing thermodynamic processes: the motion \( x \), the second Piola- Kirchhoff stress tensor \( S \), the body force per
unit mass $b$, the Helmholtz free energy $\Psi$, the specific entropy $s$, the heat supply $r_0$, the absolute temperature $T$, the heat flux vector per unit area $q$, the elastic strain tensor $E^e$, the inelastic strain tensor $E^I$ and a set of internal state variables $\alpha^{(I)}$ characterizing the material.

- The fundamental equations of mechanics postulating for the balance laws of linear momentum, angular momentum, and energy, as well as the second law of thermodynamics expressed in the variables given above.
- The constitutive assumptions describing the material response and abiding the compatibility of the constitutive equations with the fundamental equations of mechanics.

This leads to the coupled thermomechanical equation:

$$\rho C_v \dot{T} = r_0 + K \nabla^2 T - \left( \beta : 4\hat{D} : \dot{E}^e \right) T + S : \dot{E}^I$$

where $\beta$ denotes the coefficient of the thermal expansion matrix, $C_v$ (J.kg$^{-1}$K$^{-1}$; Joule per kilogramme per Kelvin degree) the specific heat at constant deformation, $4\hat{D}$ the fourth-order elasticity tensor, $e$ the specific internal energy, $K$ (W.m$^{-1}$K$^{-1}$; Watt per metre per Kelvin degree) the thermal conductivity and finally $T$ the absolute temperature. The volumetric heat capacity $C = \rho C_v$ of the material is the energy required to raise the temperature of an unit volume by 1° Celsius (or Kelvin degree). This equation shows the potential applications and various uses of the infrared scanning technique in engineering problems.

4. Fast Evaluation of Fatigue Limit

Infrared thermography is a technique for producing heat pictures from the invisible radiant energy emitted from stationary or moving objects at any distance and without surface contact or in any way influencing the actual surface temperature of the objects viewed.

4.1. Infrared Scanner

A scanning camera is used which is analogous to a television camera. It utilizes an infrared detector in a sophisticated electronics system in order to detect radiated energy and to convert it into a detailed real-time thermal picture in a video system both color and monochromatic. Response times are shorter than one microsecond. Temperature differences in the heat patterns are discernible instantly and represented by several hues. The quantity of energy $W$ (W.m$^{-2}$,µm$^{-1}$), emitted as infrared radiation, is a function of the temperature and emissivity of the specimen. The higher the temperature, the more important the emitted energy. Differences of radiated energy correspond to differences of temperature. Since the received radiation has nonlinear relationship to the object temperature, and can be affected by atmosphere damping and includes reflected radiation from objects surroundings, calibration and correction procedures have to be applied. Knowing the temperature of the reference,
the view field temperature can then be calculated with a sensitivity of 0.2 °C at 20 °C.

4.2. Rotating Bending Tests

The material for the rotating bending tests was a XC55 steel, quite extensively used in automobile construction. To minimize scatter, the 6.74 mm in diameter specimens have been prepared from a single melt. The number of load cycles was $10^7$ corresponding to the automobile fatigue damage. Cycle fatigue tests were performed on a 4-point loading rotation bending Schenck machine running at approximately 100 Hz. A total of 18 specimens was step-tested according to the Staircase or Up-and-Down method, meaning that each specimen was run for $10^7$ cycles at 370 MPa, whereupon the stress was raised in 10 MPa step if there is non-failure and the stress is lowered in case of failure. Using standard methods of statistical analysis, the fatigue limit has been estimated equal to 399 MPa with a standard deviation of 41 MPa. Series of 5 rotating bending tests have been scanned by the infrared system at different stress levels. The load duration was chosen 60 seconds corresponding to 6,000 load cycles and 3,000 load cycles at 30 seconds in duration. Each colour hue corresponds to 0.2 °C. The software TIC 8000 allowed the data reduction of the thermal images which shows heat generation after 3,000 and 6,000 load cycles. The fatigue damage mechanism is revealed by the change of intrinsic dissipation regime. Experimental results have been summarized in a (stress/intrinsic dissipation) plot where it can be seen how the fatigue limit is determined using a graphic procedure. The threshold of critical thermal dissipation is roughly the same for the 3,000 cycles and 6,000 cycles curve and it corresponds to the value deduced from standard procedure.

5. Concluding Remarks

Owing to the thermomechanical coupling, infrared thermography provides a non-destructive, noncontact and real-time test to observe the physical process of metal degradation and to detect the occurrence of intrinsic dissipation. Thus it readily gives a measure of the material damage and permits to evaluate the limit of a progressive damaging process under load beyond which the material is led to failure. It is of particular interest that the method allows not only qualitative work such as finding flaws, but also quantitative analysis of effects of flaws on strength and durability of structural components. This useful and promising technique offers accurate illustration of the initiation of a crack, the onset of its unstable propagation through the material and/or flaw coalescence when increasing irreversible microcracking is generated by vibratory loading. Particularly in case of fatigue testing, the infrared thermographic technique make evidence of the initiation of a crack and its propagation through the material. Detecting a strong change of intrinsic dissipation regime, this method readily allows a short-time evaluation of the endurance limit, commonly determined by a very time-consuming procedure.
References

The $\sqrt{\text{area}}$ Parameter Model for Prediction of Fatigue Strength of Materials containing Small Defects: Analysis, Experimental Evidences and Applications

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The importance of the influence of small defects and nonmetallic inclusions on fatigue of metals has been recognized for a long time. So many investigations have been carried out in this field that it is rather difficult to make an exact and impartial survey. The complication of the affecting mechanisms and inclusion configurations has prevented the establishment of a reliable quantitative method of evaluating the effects of small defects and nonmetallic inclusions.

In this paper, evidence is shown that small cracks, defects and nonmetallic inclusions having the same value of the square root of projection area, $\sqrt{\text{area}}$, have the same influence on the fatigue strength regardless of different stress concentration factors. The reason is explained on the basis of both microscopic observation that fatigue limit is determined by condition of nonpropagating crack and numerical analysis of cracking from small cracks, defects and inclusions.

It is reasonable to seek the geometrical parameter from the standpoint that the effects of shapes and sizes of cracks on fatigue strength may be correlated with stress intensity factors, especially with the maximum stress intensity factor $K_{I\text{max}}$ along the three-dimensional crack front. Previous numerical studies on this problem by Murakami and Nemat-Nasser, and Murakami can be summarized by the following approximate equations [1, 2]:

\[
K_{I\text{max}} = 0.65\sigma_0\sqrt{\pi\sqrt{\text{area}_s}}; \quad \text{for surface crack} \quad (1)
\]

\[
K_{I\text{max}} = 0.50\sigma_0\sqrt{\pi\sqrt{\text{area}_i}}; \quad \text{for internal crack} \quad (2)
\]

where $\sigma_0$: the maximum principal stress, $\text{area}_s$: the area of a surface crack and $\text{area}_i$: the area of an internal crack. Figure 1 shows some numerical examples.

The choice of the geometrical parameter $\sqrt{\text{area}}$ and the Vickers hardness $H_V$ enables one to predict the fatigue limit $\sigma_w$ by the proposed equations [3, 4] (see Figs. 2 and 3):

(a) Surface defects under $R = -1$:

\[
\sigma_w = 1.43(H_V + 120)/((\sqrt{\text{area}})^{1/6}) \quad (3)
\]

(b) Surface, subsurface and internal defects under $R \neq -1$:

\[
\sigma_w = C(H_V + 120)/((\sqrt{\text{area}})^{1/6} \cdot [(1 - R)/2]^{\alpha}) \quad (4)
\]

\[
\alpha = 0.226 + H_V \times 10^{-4}
\]

\[
C = 1.43 \quad \text{(surface defect)}, \quad 1.41 \quad \text{(subsurface defect)}
\]

\[
\text{and 1.56 (internal defect)}
\]

where the units are $\sigma_w$ in MPa, $H_V$ in kgf/mm$^2$ and $\sqrt{\text{area}}$ in $\mu$m, and $R$ is the $R$-ratio. The above equations are valid over a range that is dependent on the material.

Table 1 shows the application of the prediction equation to fatigue failure from nonmetallic inclusions.
Table 1: Size and location of inclusions and fatigue limit predicted by Eq.(4). (Bearing steel: \(H_V = 758\))\[4\].

<table>
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<tr>
<th>Nominal stress at surface, (\sigma) (MPa)</th>
<th>Cycles to failure, (N_f)</th>
<th>Inclusion size, (\sqrt{area}) ((\mu)m)</th>
<th>Distance from surface, (h) ((\mu)m)</th>
<th>Shape of inclusions</th>
<th>Nominal stress at inclusion, (\sigma') (MPa)</th>
<th>Fatigue limit predicted by Eq.(8), (\sigma''/\sigma'_n)</th>
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<tr>
<td>1030</td>
<td>1.25 \times 10^4</td>
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<td>556.77</td>
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<td>981</td>
<td>422.95</td>
<td>15.2</td>
<td>28</td>
<td>(\uparrow)</td>
<td>973</td>
<td>870</td>
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<td>981</td>
<td>898.01</td>
<td>9.9</td>
<td>74</td>
<td>(\uparrow)</td>
<td>963</td>
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<td>350</td>
<td>(\uparrow)</td>
<td>886</td>
<td>782</td>
</tr>
</tbody>
</table>

References

Figure 1: Relationship between the maximum stress intensity factor $K_{I\text{max}}$ and $\sqrt{\text{area}}$ for various surface cracks[1, 2].

Figure 2: Relationship between $\Delta K_{I\text{max}}$ and $\sqrt{\text{area}}$ for various defects and cracks. Letters correspond to the materials given in reference[3, 4]. Numbers in parentheses indicate the Vickers hardness $H_V$ of materials.

Figure 3: Relationship between $\sigma/(H_V + 120)$ and $\sqrt{\text{area}}$. Letters correspond to the materials given in reference[3, 4].
SESSION 6

Damage Mechanics
The present study focuses on the failure modes of brittle solids with disordered microstructures. The phenomenology of the considered failure modes is related to the irreversible stochastic processes on the atomic and microstructural scales: namely, the rupture of atomic bonds manifested by the nucleation and growth of microcracks. A rich spectrum of physical phenomena and new concepts that emerge from these stochastic studies demonstrates the limitation of the traditional, deterministic, and local continuum models of mechanics of brittle solids.

The study is limited to the materials with inferior tensile (cohesive) strength and the case when the inelastic deformation is attributable to the evolution of damage (nucleation and growth of a diffuse distribution of materials. Non-crystalline rocks, concrete, glass, ceramics, silicon, iron, thermosets and human bones belong to the class of these materials.

All failure modes are divided into intrinsic and extrinsic classes. The failure mode is intrinsic if its threshold is defined in terms of the effective material properties (constitutive laws) and micro-defect densities. Thresholds of extrinsic failure modes are extensive properties since they also depend on the specimen size and shape.

The damage evolution is related to the nucleation of new and growth of the already existing microcracks. The conditions for the onset of microcrack propagation and the stability of this propagation is defined in the traditional manner in the form of the Griffith’s relations between the elastic energy release rate $G$ and the thermodynamic force $R$ resisting the change of the crack surface area. The traditional estimates of the energy release rate $G$ are of limited utility since the microcracks are of irregular shape, almost never planar, often interfacial and embedded in a material that is neither homogeneous nor isotropic on the considered scale. The resistive force $R$ (cohesive or fracture energy) depends on the distribution of energy barriers that are strong enough to prevent or hinder the microcrack growth. The local fluctuations of these two forces have a strong influence on the pattern of the damage evolution, brittle to ductile transition and failure modes. Hence, it seems reasonable to conclude that a rational estimate of the irreversible rearrangements of the microstructure during a brittle deformation cannot always be derived using traditional, deterministic models. One of the principal objectives of this study is to define the circumstances under which the traditional methods may in fact be useful in estimation of the failure thresholds.

A complete statistical description of the microstructural disorder is rarely possible and only seldom required. This is also true in the description of critical states, i.e. states at the imminent failure. However, the critical states cannot always be defined in the terms of volume averages (densities) of random fields. The irreducible set of random (statistical) length parameters consists of two parameters which define the material and the accumulated damage; namely, (a) the characteristic length of the microstructure (grain size, etc.) and (b) the correlation length. The specimen size (and, sometimes, shape) must also be included to determine the probability of
failure in a perfectly brittle mode. The size of the representative volume element (RVE) must be determined to define the range of the applicability of continuum theories and the distance which separates pairs of consecutive signals is introduced to pinpoint the type of the damage evolution. The resolution length of a model or test device is needed to make a distinction between the defect nucleation and defect growth.

The scale on which the material is statistical homogeneous, applicability of the local continuum models, objectivity of a particular discretization of a solid into finite elements can be readily defined in terms of these characteristic lengths. Moreover, the pattern of the damage evolution and the mode of failure can be discerned from the measurements of the correlation length.

The perfectly brittle failure of a specimen in uniaxial tension is a quintessential example of an extrinsic failure mode. The failure mode depends on the extreme statistics of the pre-existing defects. The specimen fails as soon as one of these pre-existing defects becomes unstable in Griffith’s sense. The probability of brittle failure depends on the specimen volume and the distribution of the sizes of pre-existing defects. Hence, it can be written in the form of Weibull or Gumbel expression depending on whether the distribution of crack sizes is defined by a power or exponential law.

The elastic percolation, localization and tensile failure of a damage tolerant solid are taken as examples that illustrate some of the principal aspects of the intrinsic failure modes. It is further shown that the distinction between these three failure modes requires statistical consideration of the introduced characteristic lengths.

The elastic percolation failure can occur only when the damage evolution is controlled by the nucleation of microcracks (i.e. in the absence of crack growth). Comminution of rocks in three-axial compression is a typical example of a deformation mode which may lead to the percolation transition. The percolation threshold is defined as the loss of the specimen connectivity (at which the effective secant modulus vanishes) which is detected by the kinematics of the percolation cluster reflected in the rate of increase of the correlation length. The fact that the microstructural geometry in the vicinity of the percolation threshold is statistically self-similar (scale invariant) is a reason that explains the surprising accuracy of the mean field estimates of the critical state.

The statistical model of localization clearly demonstrates the limitations of the conventional, deterministic continuum models. These models also show that the micro-structural disorder is the key element of the localization process. Introduction of the disorder provides a proof that: (a) the localization is not instantaneous, (b) the onset of localization is a matter of definition and (c) the band geometry is irregular and (d) the width of the shear band (fault) depends on the type of the measurement. The material within the fault is neither homogeneous (as assumed in continuum models) nor self- similar. The band width of the fault changes along its length. Thus, it is necessary to introduce an averaging measure that is related to the transport properties. For example, a specimen with a fault can be subjected to a periodic tractions at one end of the specimen. Measurements of the energy on the opposite end provides an estimate of the lag induced that is proportional to the fault width. It was shown that the band width (i.e. the vibrational density of states) is
not constant. It’s dependence on the frequencies of the externally applied tractions
takes a form of a power law.

In summary, the present study demonstrates the fact that some of the aspects of
brittle failure modes are model induced. Despite their elegance and computational
efficiency the traditional, local continuum models cannot always be applied. The
loss of homogeneity in a majority of cases signals that the statistical aspects cannot
be ignored.
Introduction

Slurry infiltrated mat concrete (SIMCON), developed by Hackman, Farrel, and Dunham in 1992, is fabricated by placing in a mold a factory-prepared long steel fiber mat with uniform fiber distribution and infiltrating cement-based slurry with external vibration to enforce uniform slurry distribution. Laboratory test data show that SIMCON exhibits strength and ductility properties superior to slurry infiltrated fiber concrete (SIFCON) developed by Lankard in 1982. In addition, the use of pre-fabricated steel-fiber mat assures uniform fiber distribution, which is difficult to achieve in SIFCON. To date, no micromechanics-based analytical models have been reported for this high performance fiber-reinforced cementitious composite.

In this paper, the strength and ductility mechanisms of SIMCON tension members are investigated both experimentally and analytically. In addition to standard strength testing, special tests were conducted to understand the microstructural interaction between steel fibers and the cement matrix and relate the interaction to the strength and ductility of SIMCON. It was found that the ductility of SIMCON mainly stems from plastic deformation of steel fibers rather than fiber pull-out. By utilizing a two-phase mixture model, the ductility of SIMCON tension members were estimated based upon simulated force-displacement relations including linearly elastic, nonlinear hardening, and softening regimes.

Tensile Behavior of SIMCON and SIFCON

Prismatic SIMCON specimens of 305×38×25 mm were fabricated with fiber volume fractions of 3.04%, 3.33%, and 4.64%. A mat consisted of stainless steel (A430) fibers of length 142 mm and an average rectangular cross section of 0.728×0.133 mm. Cement-based slurry properties were: water/cement = 0.35, sand/cement = 0.50, and superplasticizer liquid/cement = 0.025 by weight. An overall stress-strain relation of a SIMCON tension member with 4.64% fiber volume fraction exhibits a linear response up to approximately 15 to 20% of peak stress, and then a nonlinear hardening regime up to the peak stress, followed by a softening regime. The nonlinear hardening regime is characterized by three to four multiple crack formations, while the softening regime is due to crack localization. Close observation of failed specimens revealed that bridging fibers had failed after large plastic deformation. This indicates that the ductility of SIMCON is due to plastic deformation of steel fibers.

As a comparison, SIFCON tension members with short steel fibers having a volume fraction of 4.74% were also fabricated. The dimensions of the fibers were
The measured peak stress of SIFCON was approximately 35% that of SIMCON. In the SIFCON specimens, most bridging fibers were pulled out without failure, unlike the long SIMCON fibers. In this experiment, SIMCON members exhibited superior strength and ductility to SIFCON members. However, typical SIFCON members include much higher fiber volume fractions and could exhibit higher tensile strength and ductility by inducing multiple cracking.

**Ductility Control of SIMCON Tension Members**

In order to test the ductility hypothesis, pre-cracked specimens with three bridging fibers were fabricated with various debonded fiber lengths. The length was controlled by wrapping the fibers with Teflon tape. The debonded length was set to be 0, 5.1, 10.2, and 15.2 mm on each side of the crack. If the major ductility of SIMCON is due to the plastic deformation of steel fibers, specimens with longer debonded fibers should exhibit larger ductility.

Experimental data reinforces the hypothesis that the major ductility of SIMCON is due to the plastic deformation of fibers. Therefore, the ductility of SIMCON tension members can be enhanced by weakening the fiber-cement interface to create longer debonded regions on both sides of each crack. In the experiments, it was found that the steel fibers in a SIMCON mat were notch sensitive. A continuum model was constructed by incorporating the random nature of flow distributions (weak links) and the toughness under ductile failure based upon the J-integral. The above bond experiments were simulated by utilizing the two-phase mixture bond model, developed by Hegemier, Murakami, and Hageman (1985). The model accurately simulated the experimental stress-strain relations.

**Mechanical Modeling of Inclined Fibers in SIMCON**

In order to simulate the overall stress-strain relation of SIMCON which is made of a fiber mat with a uniform planar distribution of fibers, the contribution of inclined
fibers has to be evaluated. Pre-cracked tension specimens with two symmetrically inclined fibers were fabricated. The examination of the deformation and failure of the failed bridging fibers implies that the failure is due to ductile fracture caused by localized bending stress in the exposed region of each fiber. By incorporating the effect of inclined fibers in the model, the overall stress-strain relation of the SIMCON tension test was simulated by the two-phase mixture model.

References

A Finite-Element Simulation Method for a System of Growing Cracks in a Heterogeneous Material

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In this paper a numerical simulation method is proposed for the growth of interacting cracks in a finite heterogeneous two-dimensional elastic solid. A crack growth pattern is predicted by considering the interaction of all growing cracks with the Schwartz-Neumann alternating procedure using a first order perturbation solution of a slightly branched and curved crack extension of all growing cracks[1]. The local symmetry condition at the crack tip is used as the crack path criterion. The stress analysis is based on the step-by-step finite element method using the method of superposition of analytical and finite-element solutions accompanied by the quadtree automatic mesh generation algorithm[2].

A finite and heterogeneous solid, which is composed of M sub-domains, will be considered (Figure 1). Each sub-domain consists of a linearly elastic and homogeneous solid, in which there exists no more than one crack tip. Our objective is to predict the crack growth pattern to be generated by this system. This problem can be decomposed into the two problems. The one is the problem prior to the crack growth, and the other is the problem for canceling out the disturbing tractions acting on the unknown newly created crack surfaces. The Mode I and II stress intensity factors $K_I$ and $K_{II}$ at the l-th extending crack tip can be obtained by solving the latter problem. By using the first order perturbation method with a proper order analysis for the Schwartz-Neumann alternating procedure, the $K_I^l$ and $K_{II}^l$ at the l-th extending crack tip can be expressed by the following three parts, which can be obtained by using the numerical solution of the boundary value problem prior to the crack growth and the shape parameters of the crack extension;

$$K_I^l = K_{I}^{\infty} + K_{I}^{cl} + K_{I}^{fl}, \quad K_{II}^l = K_{II}^{\infty} + K_{II}^{cl} + K_{II}^{fl}$$

where the first part is the solution of a slightly branched and curved extension of a semi-infinite crack in an infinite solid, and its expression is similar to that for a single crack problem. The second part represents effect of its own extension, and the third part, the interaction from other extending cracks. For predicting the crack growth path, the local symmetry condition is used. It is interesting to note that the expression of the curvature of the path is affected by other extending cracks.

As a numerical example, we first consider the avoidance of collinear edge cracks, which was discussed by Melin[3]. The simulated results are shown in Figure 2, where the two cracks seem to avoid with each other in the beginning. The second example is an interacting crack growth in composite materials. The two cracks initiated at the ends of fibers are shown in Figure 3. The predicted crack growth pattern is illustrated in Figure 4.

References


Figure 1: Definition of the problem
Figure 2: Avoidance of edge cracks

Figure 3: Fiber-end cracks
Figure 4: Curved interacting growth of fiber-end cracks
SESSION 7

Micromechanics Modeling I
Generalized Shape Optimization of Three-dimensional Structures using Materials with Optimum Microstructures

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This paper deals with generalized shape optimization of linearly elastic, three-dimensional continuum structures, i.e., we consider the problem of determining the design of the structural topology (or layout) such that the shape of external as well as internal boundaries and the number of inner holes are optimized simultaneously. For prescribed static loading and given boundary conditions the optimum solution is sought from the condition of maximum integral stiffness (minimum elastic compliance) subject to a specified amount of structural material within a given three-dimensional design domain.

Topology optimization of two-dimensional continuum structures was introduced in the literature by the landmark paper by Bendsøe and Kikuchi[1] in 1988 (see also Bendsøe[2]), and the present paper may be considered a generalization of this work to three-dimensional structures (see also the recent paper Allaire et al[3]). The papers[1, 2] as well as studies of different two-dimensional problems published earlier by Lurie et al[4], Cheng and Olhoff[5, 6], Olhoff et al[7] and Kohn and Strang[8], have shown that topology optimization problems are generally not well-posed unless the formulation of the problem is relaxed by introducing composites with perforated, periodic microstructures of continuously varying volume density and orientation as admissible materials for the structural design. The important point here is to choose a microstructure that allows the volume density of material to cover the complete range of values from zero (void) to one (solid), and that this microstructure is periodic so that effective material characteristics can be determined by “smear out” or homogenization.

Our determination of the microstructure of a perforated composite material with maximum effective stiffness properties subject to a volume density and any given state of stress, lends itself on the work by Gibianski and Cherkaev[9]. These authors applied the theory of quasiconvexity to construct a lower bound on the complementary energy of a composite which is valid for composites of any microstructure. At the same time, they derived the minimum energy for the class of sequential laminates of any rank, and applied this as an upper bound on energy. The optimum composites are characterized by having a complementary energy which lies between these two bounds, and it is shown in [9] that for the limiting case of perforated composites, these bounds coalesce and yield analytical expressions for the energy of the optimum perforated laminate microstructure. At the same time, explicit formulas for the parameters of the optimum microstructure are presented subject to any state of stress. Similar results have recently been obtained by Allaire[10] for three-dimensional microstructures. Based on these analytical results for optimization at the local microstructural level, the global problem of minimizing the elastic compliance for the macrostructure is easily established. The sensitivity analysis of the compliance and the total material volume of the structure can be carried out analytically, and an iterative numerical procedure based on finite element analysis, sensitivity analysis and mathematical programming has been developed for solution
of the three-dimensional, generalized shape optimization problem. Within the main loop of the iterative procedure, there is an inner loop for orienting along the axes of the local principal stresses the three-dimensional, orthotropic rank 3 laminate microstructures used, and there is subsequently another inner loop which ensures that precise correspondence is obtained between the values of the optimum lamination parameters and the principal stresses.

The numerical results show that the structures of optimum topologies obtained within the initial formulation generally turn out to consist of composites in certain subregions. In order to avoid this feature, which is less attractive from the point of view of manufacture, we may augment the mathematical formulation of the optimization problem by introducing a penalty term which penalizes formation of composite and yields solutions that entirely consist of solid, isotropic material and void.

A number of optimum three-dimensional structural topologies are presented in the paper. As a special feature, we develop some illustrative examples of full three-dimensional layout and topology optimization of plate-like structures with a view to compare the solutions with results published earlier in the literature based on the use of two-dimensional plate theories like the Kirchhoff and the Reissner-Mindlin plate theories and the more restricted classes of plate reinforcing microstructures which these theories admit.

References


Identifying Internal Variables – A Rational Approach based on Microstructures of Materials

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A natural or artificial material is usually microscopically heterogeneous and owns a certain microstructure comprising the material constitutions (crystalloid, dislocations, phase transformations, microcracks, voids, fibers, inclusions). It is not surprising that the orientation distribution function (ODF), say \( \Phi \), of the size, shape, phase, and position of the material constitutions correlates well with a number of important mechanical and physical properties of the material. Particularly, when a material undergoes inelastic deformation and heating, the accompanying evolution of observable aspects such as the ODF constitutes measurable signatures of the induced irreversible changing of the internal microstructure of the material.

It is widely recognized that thermodynamical theories of internal variables constitute the main framework in phenomenologically describing irreversible behavior (plasticity, damage, etc.) of materials. Physically, irreversible behavior of a material is due to changing microstructure of the material. A fundamental problem in internal variable theories is that how to identify the internal variables based on the material microstructure.

It has been becoming more and more clear that the concept of ODF plays a dominant role in various micromechanically based approaches of mechanical and physical properties of heterogeneous or non-homogeneous materials and provides the base for the theories of internal state variables. For example, the current stress \( \sigma(t) \) of a Noll’s simple material element depends upon the history \( F_t \) of the deformation gradient \( F \) in a functional form: \( \sigma(t) = \mathcal{G}(F_t) \). Suppose that one or a few ODFs \( \Phi(t), ..., \Psi(t) \) of the evolutional microstructures could represent the effect of the past deformation history \( F_t \). Then:

\[
\begin{align*}
\text{constitutive equation} & \quad \sigma(t) = \mathcal{G}(F(t), \Phi(t), ..., \Psi(t)), \\
\text{evolutional equation} & \quad \dot{\Phi}(t) = \mathcal{M}(F(t), \Phi(t), ..., \Psi(t)), ...,
\end{align*}
\]

where a superposed dot denotes the time derivative, and \( \mathcal{G} \) and \( \mathcal{M}, ..., \mathcal{N} \) are 2nd order tensor-valued function and scalar-valued functions, respectively. Then the convergence of the irreducible tensorial Fourier expansions of \( \Phi, ..., \Psi \) makes it possible to employ, within a desired degree of accuracy, the leading irreducible tensorial coefficients \( S_1, ..., S_n \) to represent \( \Phi, ..., \Psi \) themselves. As these irreducible tensorial coefficients act as internal state variables, the constitutive and evolutional equations are converted to the following tensor function representations:

\[
\begin{align*}
\sigma & = \Psi(F, S_1, ..., S_n), \\
DS_i/Dt & = \Xi_i(F, S_1, ..., S_n), \quad (i = 1, 2, ..., n)
\end{align*}
\]

where \( D/Dt \) denotes an objective time derivative. It has been widely accepted that the above tensor function representations in the framework of internal state variables are more convenient than functional representations, in particular for theories of elastoplasticity and damage.
There are countless physical examples of ODFs. For instance, we define the ODF $\Phi(n)$ for a planar semicrystal polymer so that $\Psi(n)\,d\varphi$ is the probability that a crystal chain axis lies in the infinitesimal sector $d\varphi$ centered at the direction $n$ or the angle $\varphi$. Similar definitions can be made for: (i) composite materials reinforced with short distributed fibers, (ii) isotropic matrix damaged due to penny-shaped microcracks, and (iii) isotropic matrix reinforced with spheroidal inclusions or weakened due to spheroidal voids. One may note that the common peculiarity of the above examples is that the geometric shapes of the crystal chains, microcracks, inclusions, and voids are all axisymmetric. For a polycrystal in three dimensions, whose single crystals could possess one of the thirty-two crystallographic groups, the single crystals are not axially symmetric. The orientation of a typical crystal $c$ within the polycrystal is described in terms of a rotation tensor $R$, and the volume fraction of crystals with the orientation $R$ can be defined as the ODF, say $\Phi(R)$.

A thorough investigation is made on the Fourier expansions with irreducible tensorial coefficients for ODFs defined on the unit sphere (i.e., for directional ODFs) and on the rotation group (i.e., for crystal ODFs). Furthermore, a general method of determining the contributions of ODFs to various linear physical properties (e.g., elasticity, piezoelectricity) are proposed. For example, it can be easily shown that for a stiffness-dominated damage only the scalar, second-order, and fourth-order irreducible tensorial coefficients of the ODFs could be effective. This ensures that elasticity damage tensors in general consist of two second-order symmetric tensors and one fourth-order irreducible tensor in three dimensions, or a single second-order symmetric tensor and a single fourth-order irreducible tensor in two dimensions. We also relate the shapes of the defects (microcracks or microvoids) to these internal tensorial variables. Particularly, with respect to various defect shapes we have identified the elasticity damage tensors.
We consider a composite, in which only a matrix, surrounding elastic inclusions, can creep. Mori, Huang and Taya (To appear in Acta Metallurgica) have shown that the creep of a composite eventually terminates when sliding and diffusion on matrix/inclusion interfaces are not allowed. Thus, the continuation of creep of a composite requires the simultaneous operation of the interfacial sliding and diffusion. We have found that stationary creep can be neatly analyzed for the following modes of deformation:

1. Circular cylindrical inclusions parallel to the $x_3$-axis and an overall external shear stress $\sigma_{12}^A = \sigma^A$ and the creep rate $\dot{\varepsilon}_{12}^* = \dot{\varepsilon}^*$.

2. Spherical inclusions, an overall uniaxial tensile stress $\sigma_{33}^A = \sigma^A$ and the creep rate of $\dot{\varepsilon}_{33}^* = \dot{\varepsilon}^* = \dot{\varepsilon}_{11}^* = \dot{\varepsilon}_{22}^* = -\dot{\varepsilon}^*/2$.

3. Circular cylindrical inclusions, an overall uniaxial tensile stress $\sigma_{33}^A$ parallel to the inclusion axis and the creep rate of $\dot{\varepsilon}_{33}^* = \dot{\varepsilon}^* = \dot{\varepsilon}_{11}^* = \dot{\varepsilon}_{22}^* = -\dot{\varepsilon}^*/2$.

Diffusion on a matrix/inclusion interface causes a jump $\dot{b}_N$ in the normal component of a displacement rate on the interface. $\dot{b}_N$ is related to the gradient of a normal force $T_N$ (per unit area) on the interface. Similarly, sliding causes a jump $\dot{b}_T$ in the tangential component of a displacement rate on the interface. $\dot{b}_T$ is proportional to a tangential force $T_T$ (per unit area) on the interface. In a stationary state, $\dot{\varepsilon}^*$ must be equal to $-\varepsilon^*_{ij} x_j$. Thus, we have a differential equation of $T_N$ with respect to an appropriate coordinate. The equation can be solved analytically and $T_N$ is found proportional to $\dot{\varepsilon}^*$. $T_N$ is linearly related to $\dot{\varepsilon}^*$. From $T$ and $x$, the average stress $\langle \sigma \rangle_\Omega$ in the inclusion is calculated, proportional to $\dot{\varepsilon}^*$. The Norton creep law is assumed for the matrix, e.g. $\dot{\varepsilon}^* = \alpha \left( \langle \sigma \rangle_M / \mu \right)^n$ with a temperature dependent term $\alpha$ and an appropriate elastic constant $\mu$ of the matrix. In the stationary state, $\dot{\varepsilon}^*$ and, thus, the stress $\langle \sigma \rangle_M$ in the matrix are uniform.

$$\sigma^A = f \langle \sigma \rangle_\Omega + (1 - f) \langle \sigma \rangle_M$$

is always valid. Here, $f$ is the volume fraction of the inclusions. Since $\langle \sigma \rangle_\Omega$ is written as a linear function of $\dot{\varepsilon}^*$ and $\dot{\varepsilon}^*$ is written in terms of $\langle \sigma \rangle_M$ (creep law), Eq.(1) gives the relationship between the external stress and stationary creep rate of a composite. That is,

$$\sigma^A = f \left( \dot{\varepsilon}^*/\beta \right) + (1 - f) \mu \left( \dot{\varepsilon}^*/\alpha \right)^{1/n}$$

Here, $\beta$ is written as

$$\beta = \begin{cases} 4c \left( kT a^3 / 4D\Omega + \eta a \right)^{-1}, & \text{Case 1} \\ 20c \left( kT a^3 / D\Omega + 3\eta a \right)^{-1}, & \text{Case 2} \\ 48c \left[ a(4\ell + 3a)kT / D\Omega \\
+ \{ 12 + 8(3 + 2\ell/a)(\ell/a) + 3(a/\ell) \} \eta \right]^{-1}, & \text{Case 3} \end{cases}$$
2c is the thickness of a hypothetical interface layer, \( k \) the Boltzmann constant, \( T \) the temperature, \( D \) the interfacial diffusivity, \( \Omega \) the atomic volume, \( \eta \) the interfacial viscosity, \( a \) the radius of the spherical or cylindrical inclusions and \( 2\ell \) the length of the cylindrical inclusions.

From Eqs. (2) to (5), we can see that inclusions accelerate creep in the small regime of the stress. Small inclusions promote this behavior. In the high stress regime, inclusions retard creep.
SESSION 8

Micromechanics Modeling II
Micromechanical Modeling of Braided Textile Composites

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full paper submitted
Experimental Adiabatic Shear Band in WHA subjected to High-strain-rate Compression

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In a ballistic environment, the deformation of a conventional DU (depleted uranium) penetrator quickly localizes into intense shear bands. The adiabatic shear behavior of uranium promotes its penetrability by flaking off and minimizing the size of the impact nosecone on the penetrator. Therefore, the uranium projectile would penetrate more efficiently with achieving a greater penetration depth. Unfortunately the DU alloy causes environmentally hazardous after launched to the target, and there is a need to replace the DU penetrators with the environmentally safe materials.

Current efforts are under way to develop a newly improved WHA penetrators with a matrix phase susceptible to adiabatic shear high strength, high hardness, modest to high ductility, and easy machinability. This paper deals with a portion of the research efforts focusing on developing high-performance tungsten composites and showing the microscopic findings of shear band and its effect on the material behavior of the WHA.

WHA chosen for this study consists of two phases; W-phase and Fe-Ni phase. The W-Fe-Ni alloy contains 93 wt pct W, and nickel and iron in a weight ratio of 7 to 3. The spheroidal tungsten particles are surrounded by a continuous Fe-Ni matrix phase. A large number of cylindrical specimens, measuring nominally 0.20” in diameter and 0.15” in length, were fabricated from the W-Fe-Ni rod for quasi-static low strain rate and dynamic high-strain-rate compression. Low strain-rate test \( (10^{-3} \text{ /sec}) \) was achieved using the conventional loading frame, whereas constant high-strain-rate test \( (> 10^3 \text{ /sec}) \) was carried out by UCSD’s recovery Hopkinson compression technique producing a tailored single dynamic pulse (Nemat-Nasser et al., Proc. R. Soc. Lond. A, Vol. 435, 371-391, 1991).

Static and dynamic material properties were measured. Microhardness tests were performed on the Fe-Ni matrix, W-Fe-Ni phase and tungsten grain, using a diamond pyramidal indenter. Surface temperature of the specimen was rigorously measured with high-speed infrared detector, developed at the UCSD. To directly quantify the shear band, all postshocked specimens were sectioned (in diametrical direction parallel to the loading direction) into rectangular plates using a wire EDM. The sectioned plates were impregnated with epoxy and ground flat for further analyses.

Experimental results show that the binder-phase (Fe-Ni) and tungsten particles have the high work-hardening rate at low-strain-rate compression. This combined work deviates from the high-strain-rate compression. The post-yielding difference between quasi-static and dynamic tests suggests that plastic work is converted into the local heat during ‘excessive’ plastic deformation. This localized adiabatic heat increases the local temperature and results in decreasing the yield stress of the WHA materials. In addition, this material is relatively insensitive to strain rate in the range of 3000 - 5000 /sec, but thermal softening behavior is dominant. Significant
decrease in ultrasonic velocities was measured with extension of shear band. The hardness increases across the shear band zone and decreases outside the zone.

With the constant strain rate, no shear band was observed under 30% compressive strain. Shear band initiates at the edge of the 40%-strained specimen without crack development and extends through the specimen thickness forming a cone shape inclined at an angle 55° to 60° to the loading direction. The extensive shear band was formed at the 50% strain with significant crack formation along the shear band zone. The microscopic analysis by SEM reveals that most fractures in the shear band are intergranular where the Fe-Ni matrix has the least resistance to fracture. However, transgranular fracture is also observed, but not significantly. It appears that W-particles take over all the deformation showing extensive elongation relative to the binder-phase. This means that tungsten particles absorb all the heat generated during deformation including the binder-phase heat. This heat appears to be confined within the tungsten particles, and the elevated temperature stimulates the elongation of tungsten particle deformation. Since tungsten has a much lower heat capacity than either iron or nickel, the binding-phase may not contribute to the process of the bulk deformation.

The W-particles seem to be acting as a “heat distributor”. But it has a much greater flow stress. Therefore, once tungsten begins to deform inelastically, it quickly heats up, making it easier to participate in the overall deformation of intense, localized shear bands. This would be one of the reasons why work-hardening behavior appears insignificant under the high-strain-rate deformation.
A Micromechanical Model for the Initial Rearrangement Stage of Liquid Phase Sintering

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Liquid phase sintering (LPS) is a powder processing method widely used in both powder metallurgy and ceramics industry. German (1985), defines LPS as sintering involving a coexisting liquid and particulate solid during some part of the thermal cycle. The liquid phase can enhance the rate of interparticle bonding and of the densification of the powder system. The most common method of obtaining a liquid phase is to use mixed powders of different materials. The liquid phase can then result from melting of the component powder with a lower melting point. Immediately after liquid formation, there is a rapid initial densification due to the capillary force exerted by the wetting liquid on the solid particles. During this initial rearrangement stage, the powder system responds as a viscous solid to the capillary action. At a later stage of LPS, as the densification rate decreases, solubility and diffusivity effects lead to grain growth, shape accommodation, and microstructural coarsening. The densification finally stops when the rigidity of the solid skeleton formed prevents further rearrangement of particles.

A constitutive model for powders in the initial rearrangement stage of liquid phase sintering (LPS), where capillary force is the driving force for densification, is developed in this paper. The formulation is based on certain theories in micromechanics of granular materials (see, Mehrabadi, et al., 1992; Nemat-Nasser and Mehrabadi, 1984; Christoffersen, et al., 1981). In these theories, a granular mass is modeled as a continuum whose material points are endowed with the overall macroscopic characteristics of a typical sample which contains a representative set of granules. Descriptions are then obtained for (i) the overall macroscopic stresses in terms of contact forces and relevant geometric measures of the microstructure; (ii) the overall incremental deformation in terms of the local relative sliding of the granules; and (iii) the overall stress in terms of the overall deformation. Similarly, in this paper, the anisotropic behavior of powder system is analyzed by considering the behavior of interparticle force and fabric which is represented by the distribution of contact normals. The proposed model is used to determine the dependence of the overall volume change on such factors as local liquid volume, uniformity of liquid distribution, contact angle, initial particle distance, initial confining pressure, particle size and viscosity. The numerical and theoretical analyses in this paper indicate that small particle size, small contact angle, low viscosity, even distribution of liquid phase and loose green powder mass enhance the amount of volume change in the initial stage of LPS. It is also found that a higher confining pressure increases the densification rate but it lowers the amount of net volume shrinkage at the end of the initial stage. The anisotropic behavior of the sample is analyzed by studying the interparticle force change and fabric change during the sintering. The results obtained concerning the effects of particle size, contact angle, temperature and green density on densification behavior are consistent with the observed behavior (German, 1985) of a powder mass. The theory developed in this paper provides a basis
for optimizing the microstructure and thereby the mechanical properties of sintered materials.

Numerical results are presented to analyze the influence of several parameters on the macroscopic volume change. These parameters are the local liquid bridge volume ratio, initial bridge contact concentration (representing uniformity of the liquid distribution), contact angle, initial particle distance (representing the green density), initial confining pressure, and particle size and viscosities (representing the temperature effects). According to the numerical and theoretical analyses presented here, small particle size, small contact angle, high temperature, even distribution of liquid phase and loose green powder mass enhance the amount of volume shrinkage in the initial stage of liquid phase sintering. Although, higher confining pressure increases the densification rate, it lowers the amount of net volume shrinkage at the end of the initial stage. The results obtained concerning the effects of particle size, contact angle, temperature and green density on densification behavior are consistent with the observed behavior (German, 1985; Kingery, 1959; Kingery, et al., 1961; Huppmann and Riegger, 1975) of a powder mass. In particular, the importance of evenly distributing the liquid phase is shown by the numerical results presented here. The anisotropic behavior of powder mass is analyzed by studying the evolution of interparticle force and fabric. The insight gained is useful in controlling and predicting the macroscopic material properties such as density, strength, toughness, elastic modulus, etc.

References

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PROGRAM
August 21, Wednesday

7:00–9:00 p.m. Welcome Party and Pre-registration at Washington Hotel 2

August 22, Thursday

9:00–9:10 Opening Session: Presider, Professor H. Abé
9:10–10:50 Session 1 Micromechanics Modeling of Composites
10:50–11:10 Coffee Break
11:10–12:40 Session 2 Smart Materials and Structures
12:40–1:30 Lunch Break
1:30–3:10 Session 3 Modeling of Inelastic Behavior-I
3:10–3:30 Coffee Break
3:30–6:00 Session 4 Modeling of Inelastic Behavior-II

August 23, Friday

9:00–10:40 Session 5 Experimental-Numerical Analysis
10:40–11:00 Coffee Break
10:40–12:40 Session 6 Damage Mechanics
12:40–1:30 Lunch Break
1:30–3:10 Session 7 Micromechanics Modeling I
3:10–3:30 Coffee Break
3:30–5:20 Session 8 Micromechanics Modeling II
5:20–5:30 Closing Session Presider, Professor M. Taya
6:00–9:00 Banquet Dinner at Washington Hotel 2